Math 72 : 4.3 Systems of Linear Inequalities in 2 variables

objectives

1) Determine if an ordered pair is a solution of one linear inequality
2) Graph a linear inequality.
3) Determine if an ordered pair is a solution of two (or more) inequalities in a system
4) Graph a system of linear inequalities.

Objectives 1) and 2) refer to a problem like:

Graph $4x - 5y \geq -20$

Objectives 3) and 4) refer to a problem like:

Graph $\begin{cases} 2x + y < 7 \\ 7x - 2y > 4 \end{cases}$

① Determine if the given points are solutions of $2x + y \le 9$.

yes a) (3,5)

step 1: Substitute $x=3$ and $y=5$

$$2(3) + 5 \le 9.$$

step 2: Use order of operations to evaluate LHS (and RHS).

$$6 + 5 \le 9$$
$$11 \le 9$$

step 3: If true result, yes.
If false result, no.

$11 \le 9$ is false because 11 is not less than or equal to 9.

NO.

yes b) (1,3)

$$2(1) + 3 \le 9$$
$$2 + 3 \le 9$$
$$5 \le 9$$

true

YES

no c) (4, -1)

$$2(4) + (-1) \le 9$$
$$8 + 1 \le 9$$
$$9 \le 9$$

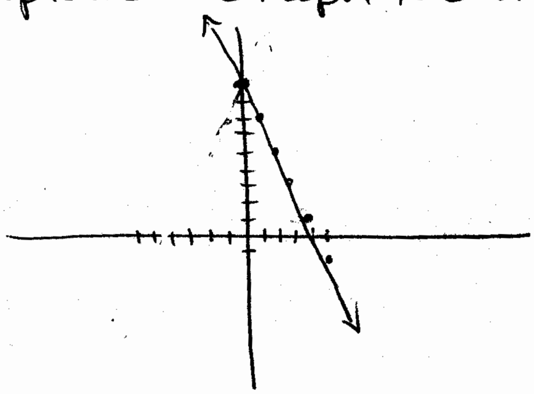
true

YES

② Explore: a) Graph the line $2x + y = 9$. (no $\le$).

no

$$y = -2x + 9$$



b) Find coordinates of 3 points on $2x + y = 9$.

(0, 9)
(1, 7)
(2, 5).

② Explore, con't.

no

c) Are the points in b) solutions of $2x+y \le 9$?

$2(0)+9 \le 9$
$9 \le 9$ yes. (0,9)
$2(+1)+7 \le 9$
$9 \le 9$ yes (1,7)
$2(2)+5 \le 9$
$9 \le 9$ yes (2,5).

d) Are the points in b) solutions of $2x+y<9$?

$2(0)+9<9$
$9<9$ no (0,9)
$2(1)+7<9$
$9<9$ no (1,7)
$2(2)+5<9$
$9<9$ no (2,5).

e) Find 3 points up and to right of $2x+y=9$. (use graph).

(5,5)
(6,0)
(0,11)

f) Are the points in e) solutions of $2x+y \le 9$?

$2(5)+5 \le 9$
$15 \le 9$ no (5,5)
$2(6)+0 \le 9$
$12 \le 9$ no (6,0)
$2(0)+11 \le 9$
$11 \le 9$ no (0,11).

g) Find 3 points down and left of $2x+y=9$ (use graph)

(0,0)
(−2,2)
(2,−3).

h) Are the points in g) solutions of $2x+y \le 9$?

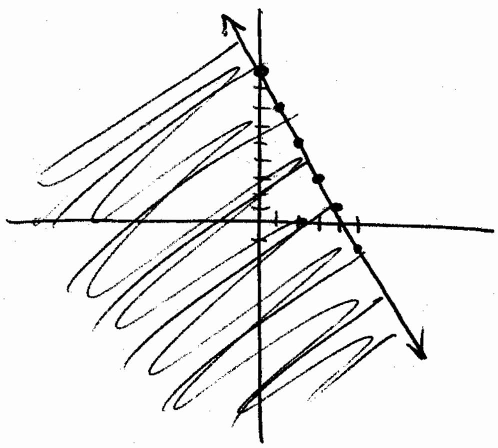
$2(0)+0 \le 9$
$0 \le 9$ yes (0,0)
$2(-2)+2 \le 9$
$-2 \le 9$ yes (−2,2)
$2(2)+(-3) \le 9$
$1 \le 9$ yes (2,−3).

no

i) graph the solutions of $2x+y \le 9$

step 1: Draw line $2x+y=9$. (solid line)

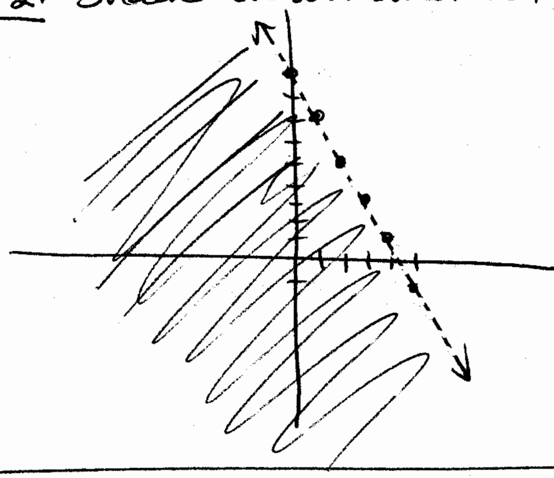
step 2: Shade down and left of the line.



j) graph the solutions of $2x+y<9$.

step 1: Draw line $2x+y=9$ (dotted line)

step 2: Shade down and left of line.



yes

To graph a linear inequality

step 1: Use solid lines for $\le$ or $\ge$.
Use dotted lines for $<$ or $>$.

step 2: Change $\le, \ge, <,$ or $>$ to $=$ and graph line.

step 3: Test a point that's not on the line, plot test point.
If test point is true, shade side including test point.
If test point is false, shade side that does not include the test point.

Alternate step 3: Isolate y in inequality if $y>$ or $y\ge$ shade up
if $y<$ or $y\le$ shade down

yes ③ Graph $y < 2x - 5$.

<u>step 1</u>: $<$ means dotted.

<u>step 2</u>: Graph $y = 2x - 5$

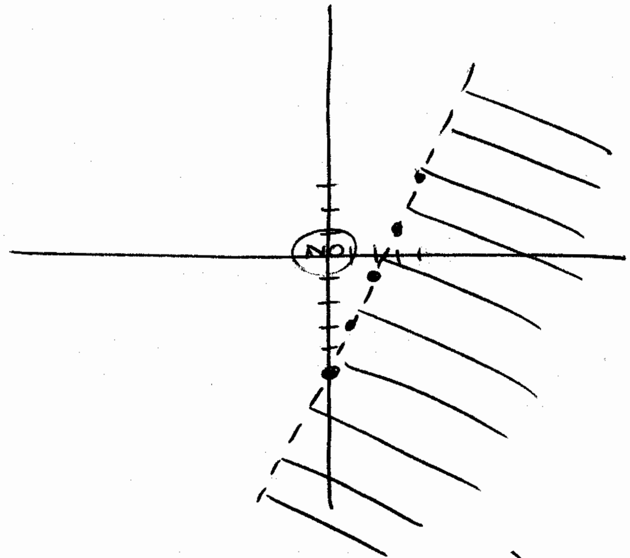
$m = 2$

y-int $(0, -5)$

<u>step 3</u>: Test $(0,0)$

$0 < 2(0) - 5$

$0 < -5$ false



yes ④ Graph $-4x + 3y \geq 0$

<u>step 1</u>: $\geq$ means solid line.

<u>step 2</u>: $-4x + 3y = 0$

$3y = 4x$

$y = \frac{4}{3}x + 0$

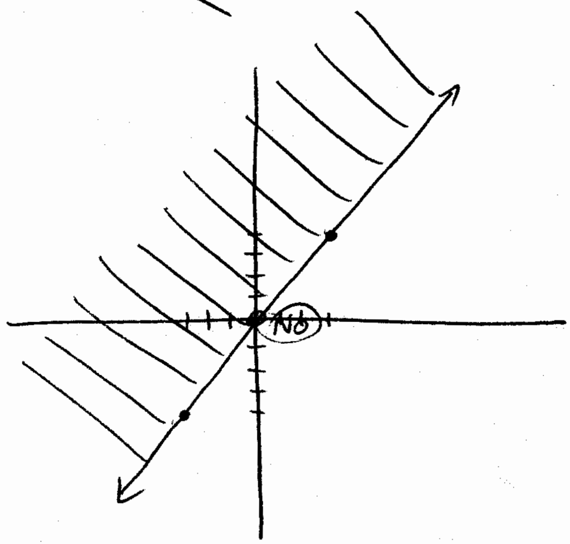
$m = \frac{4}{3}$

y-int $= (0,0)$

test $(1,0)$

$-4(1) + 3(0) \geq 0$

$-4 \geq 0$ false



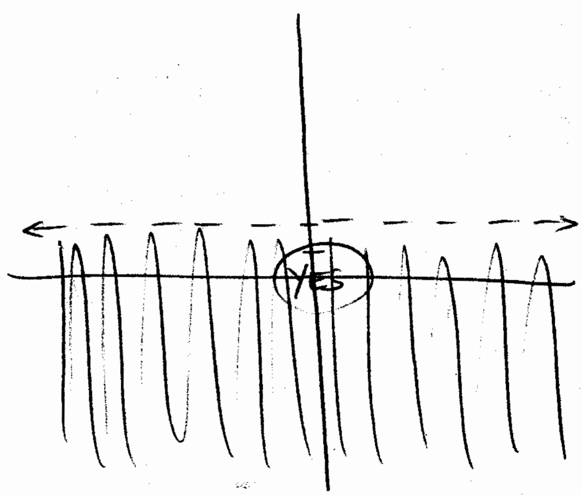
no ⑤ Graph $y < 2$.

<u>step 1</u>: $<$ means dotted

<u>step 2</u>: $y = 2$ horizontal.

<u>step 3</u>: Test $(0,0)$

$0 < 2$ yes

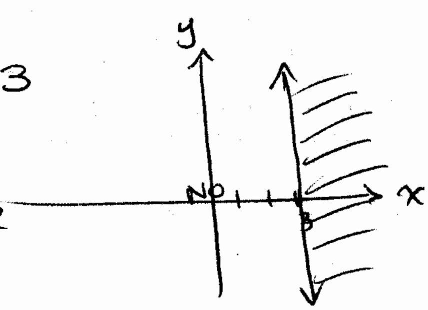


no

⑥ Graph $x \geq 3$

test $(0,0)$

$0 \geq 0$ false



yes

(7)

Graph the inequality $4x + 5y \geq -20$.

Use the graphing tool to graph the inequality.

Isolate y:

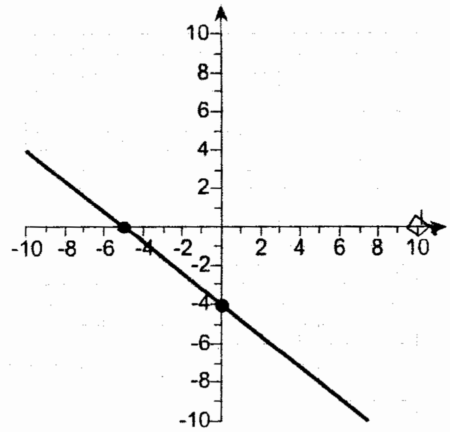
$5y \geq -4x - 20$

$y \geq -\frac{4}{5}x - 4$

slope $m = -\frac{4}{5}$

y-int $(0, -4)$

$\geq$ solid, shade upward



⑦ Kevin received \$2 from Grandma and went to a candy store where suckers are \$0.20 and taffy is \$0.25.

a) Write a linear inequality showing how many suckers (x) and how many taffies (y) Kevin can buy?

cost of suckers + cost of taffy

$$.20x + .25y \leq 2$$

$.20$ (cost per sucker) $\cdot$ x (# of suckers) + $.25$ (cost per taffy) $\cdot$ y (# of taffies) $\leq$ 2 (↑ available money)

$$\boxed{.2x + .25y \leq 2}$$

b) Graph the inequality.

step 1: $\leq$ means solid line

step 2: graph $.2x + .25y = 2$

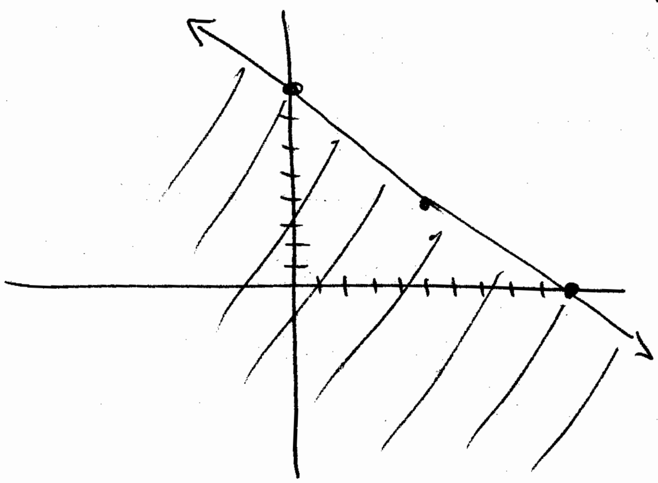
mult by 100 $\quad \frac{20x}{5} + \frac{25y}{5} = \frac{200}{5}$

$\div 5$

$$4x + 5y = 40$$

$$\frac{5y}{5} = \frac{-4x}{5} + \frac{40}{5}$$

$$y = \frac{-4}{5}x + 8$$



c) Can Kevin buy 6 suckers and 3 taffies?

$$.2(6) + .25(3) \leq 2$$

$$1.95 \leq 2 \quad \boxed{\text{yes}}$$

d) Can Kevin buy 5 suckers and 5 taffies?

$$.2(5) + .25(5) \leq 2$$

$$2.25 \leq 2 \quad \boxed{\text{no}}$$

⑧ A kindergarten class has \$120 maximum to spend going to the aquarium. Students cost \$3, adults \$5.

a) Write a linear inequality showing the number of students, x, and # of adults, y, that can go.

$$3x + 5y \le 120$$

$\begin{pmatrix}\text{cost per student}\end{pmatrix} \cdot \begin{pmatrix}\#\text{ students}\end{pmatrix}$; $\begin{pmatrix}\text{cost per adult}\end{pmatrix} \cdot \begin{pmatrix}\#\text{ adults}\end{pmatrix}$; $\le$: less than or equal to ; 120: available money

b) Can 32 students and 6 adults go?

$$3(32) + 5(6) \le 120$$
$$126 \le 120 \quad \boxed{\text{no}}$$

c) Can 29 students and 4 adults go?

$$3(29) + 5(4) \le 120$$
$$107 \le 120 \quad \boxed{\text{yes}}$$

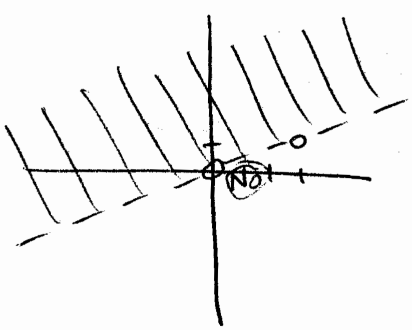
Graph the inequalities

⑨ $y > \frac{x}{3}$

$> \Rightarrow$ dotted

$y = \frac{x}{3}$

$y = \frac{1}{3}x + 0$



Test (1, 0)

$0 > \frac{1}{3}$ false

⑩ $\frac{x}{4} + \frac{y}{2} < 1$

$< \Rightarrow$ dotted

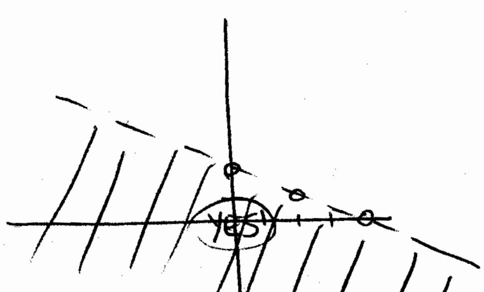
$\frac{x}{4} + \frac{y}{2} \cancel{\le} 1$

$4 \cdot \frac{x}{4} + 4 \cdot \frac{y}{2} = 1 \cdot 4$

$x + 2y = 4$

$2y = -x + 4$

$y = -\frac{1}{2}x + 2$



Test (0,0)

$\frac{0}{4} + \frac{0}{2} < 1$

$0 < 1$ true

⑪ $-3 \ge x - y$

is equivalent to

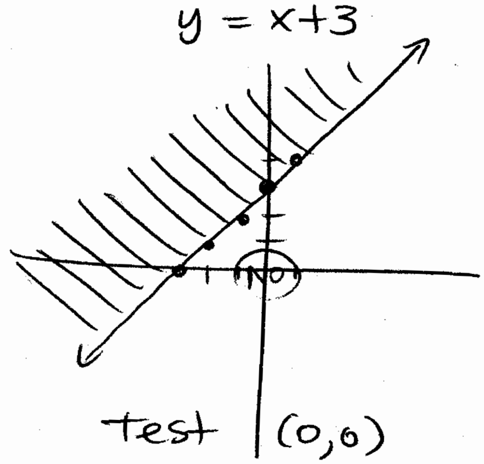
$x - y \le -3$.

$\le \Rightarrow$ solid

$x - y = -3$

$-y = -x - 3$

$y = x + 3$



Test (0,0)

$-3 \ge 0 - 0$

$-3 \ge 0$ false

yes ① Determine if (2,1) is a solution of $\begin{cases} 2x+y \le 7 \\ 7x-2y \ge 4. \end{cases}$

Step 1: substitute given point into first inequality.
If true result, go on to step 2.
If false result, answer is no.

$2(2)+(1) \le 7$
$4+1 \le 7$
$5 \le 7$ true, continue

Step 2: substitute given point into second inequality
If true result, yes.
If false result, no.

$7(2)-2(1) \ge 4$
$14-2 \ge 4$
$12 \ge 4$ true

YES, (2,1) is a solution of this system

yes ② Determine if (−2,5) is a solution of $\begin{cases} 2x+y \le 7 \\ 7x-2y \ge 4 \end{cases}$

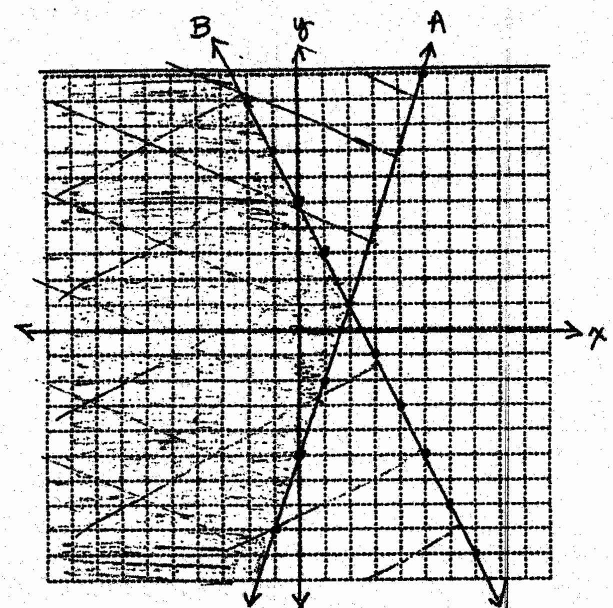
Step 1: $2(-2)+5 \le 7$
$-4+5 \le 7$
$1 \le 7$ true, continue.

Step 2: $7(-2)-2(5) \ge 4$
$-14-10 \ge 4$
$-24 \ge 4$ false

No, (−2,5) is not a solution of this system

Just as with systems of linear equations, the solution to a system of linear inequalities must be a solution of all inequalities in the system.

no ③ Graph $\begin{cases} y \geq 3x - 5 & A \\ y \leq -2x + 5 & B \end{cases}$



step 1: Graph A.

A: $\geq$ solid line

slope 3

y-int −5

test (0,0): $0 \geq 3(0) - 5$

$0 \geq -5$ true

step 2: Graph B.

B: $\leq$ solid line

slope −2

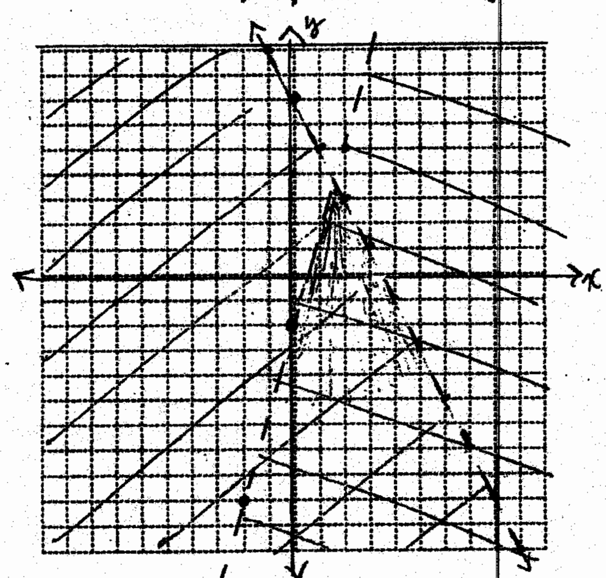
y-int 5

test (0,0): $0 \leq -2(0) + 5$

$0 \leq 5$ true

step 3: Shade the overlap.

yes ④ Graph $\begin{cases} 2x + y < 7 & A \\ 7x - 2y > 4 & B \end{cases}$



step 1: Graph A

$2x + y < 7$

$y < -2x + 7$

$\begin{cases} < \text{ dotted} \\ m = -2 \\ \text{y-int } 7 \end{cases}$

test (0,0):

$2(0) + 0 < 7$

$0 < 7$ true

step 2: Graph B.

$7x - 2y > 4$

$\frac{-2y}{-2} > \frac{-7x}{-2} + \frac{4}{-2}$

$y < \frac{7}{2}x - 2$

$\begin{cases} > \text{ dotted} \\ m = \frac{7}{2} \\ \text{y-int } -2 \end{cases}$

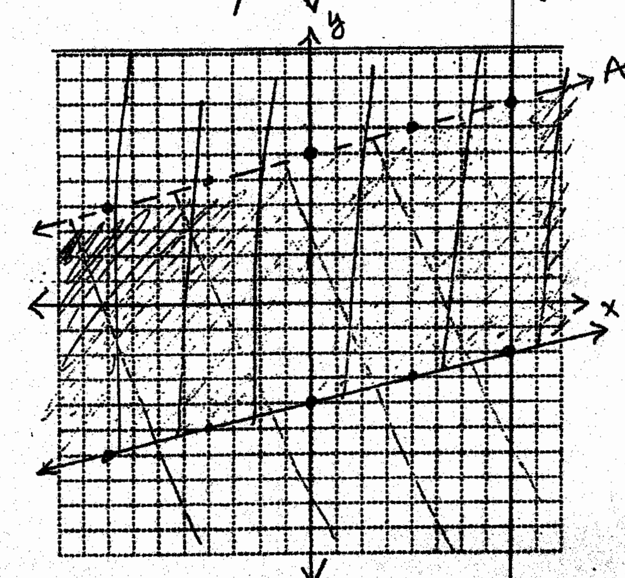
test (0,0)

$7(0) - 2(0) > 4$

$0 > 4$ false.

step 3: Shade the overlap.

yes ⑤ Graph $\begin{cases} y < \frac{1}{4}x + 6 & A \\ y \geq \frac{1}{4}x - 4 & B \end{cases}$



A: dotted, slope ¼, y-int 6

$0 < \frac{1}{4}(0) + 6$

$0 < 6$ true

B: solid, slope ¼, y-int −4

$0 \geq \frac{1}{4}(0) - 4$

$0 \geq -4$ true

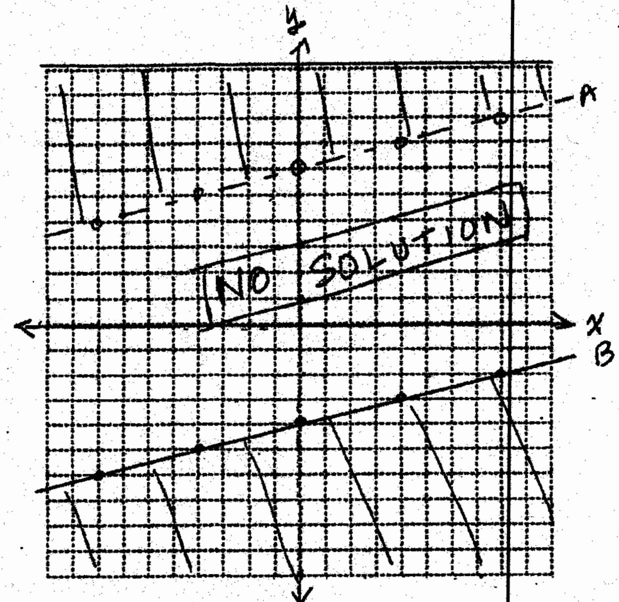
shade overlap

yes ⑥ Graph $\begin{cases} y > \frac{1}{4}x + 6 & A \\ y \le \frac{1}{4}x - 4 & B \end{cases}$

A: dotted, slope $\frac{1}{4}$, y-int 6.
test (0,0) $0 > \frac{1}{4}(0) + 6$
$0 > 6$ false

B: solid, slope $\frac{1}{4}$, y-int -4
test (0,0) $0 \le \frac{1}{4}(0) - 4$
$0 \le -4$ false

NO OVERLAP! So **no solution**.
There are no ordered pairs that make both inequalities true.

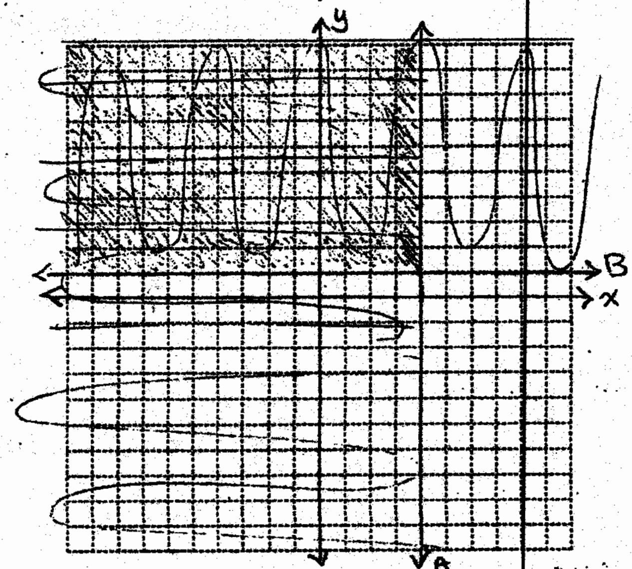


yes ⑦ Graph $\begin{cases} x \le 4 \\ y > 1 \end{cases}$

Step 1: $x \le 4$ vertical, solid
test (0,0) $0 \le 4$ true

Step 2: $y > 1$ horizontal, dotted
test (0,0) $0 > 1$ false

Step 3: shade overlap



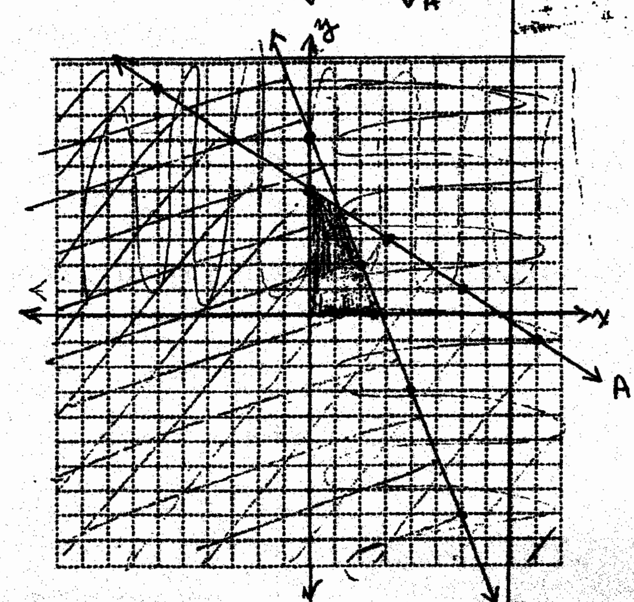
time permitting ⑧ Graph $\begin{cases} y \le -\frac{2}{3}x + 5 & A \\ y \le -\frac{5}{2}x + 7 & B \\ x \ge 0 & C \\ y \ge 0 & D \end{cases}$

A: solid, $m = -2/3$, y-int 5
$0 \le -2/3(0) + 5$ $0 \le 5$ true

B: solid, $m = -5/2$, y-int 7
$0 \le -5/2(0) + 7$ $0 \le 7$ true

C: solid, vertical, same as y-axis.
test (1,0) $1 \ge 0$ true

D: solid: horizontal, same as x-axis
test (0,1) $1 \ge 0$ true



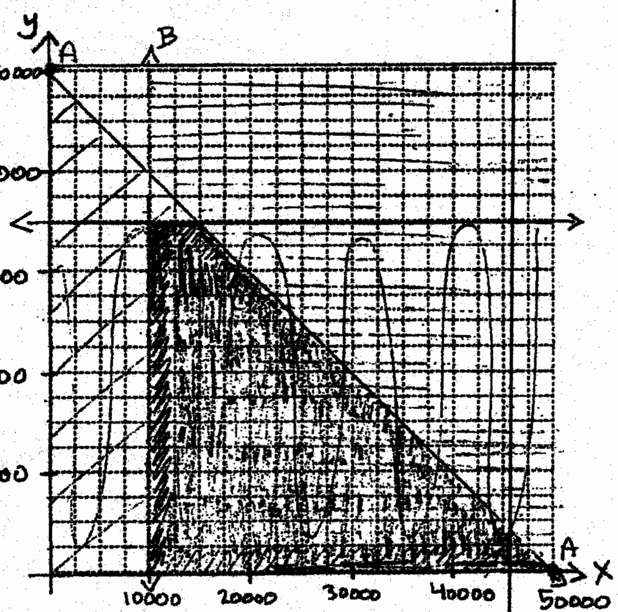
no ⑨ You have up to \$50,000 to invest. Your advisor says to put at least \$10,000 in Treasury notes (x), and no more than \$35,000 in corporate bonds (y). This gives the system of linear inequalities

$$\begin{cases} x+y \le 50{,}000 & A \\ x \ge 10{,}000 & B \\ y \le 35{,}000 & C \end{cases}$$

a) Graph the system of inequalities
b) Can you put \$20,000 in bonds and \$30,000 in treasuries?
c) Can you put \$40,000 in bonds and \$10,000 in treasuries?

a) Negative values of x or y do not make sense, so we draw only QI.

A: $x+y \le 50{,}000$
$y \le -x+50{,}000$
slope -1, y-int. 50000.
x-int: 50,000 also.
test (0,0) $0+0 \le 50000$ true



B: $x \ge 10{,}000$
vertical, solid.
test (0,0) $0 \ge 10{,}000$ false

C: $y \le 35{,}000$
horizontal solid.
test (0,0) $0 \le 35{,}000$ true

b) $x = 30000$, $y = 20000$

$20000 + 30000 \le 50000$ ✓
$30000 \ge 10000$ ✓
$20000 \le 35000$ ✓

YES

c) $x = 10{,}000$, $y = 40000$

$10000 + 40000 \le 50000$ ✓
$10000 \ge 10000$ ✓
$40000 \le 35000$ ✗

NO

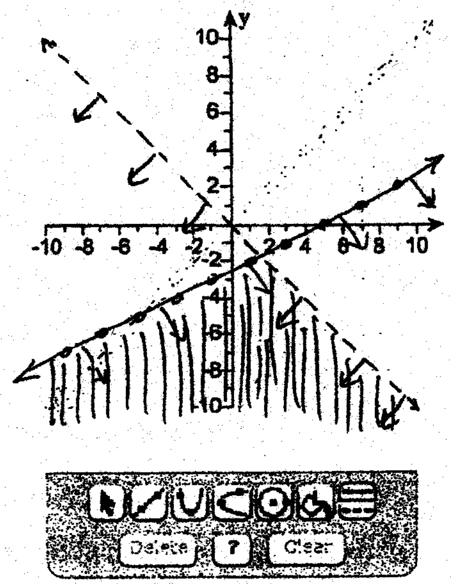
yes (10)

Graph the system of linear inequalities.

$$\begin{cases} x \geq 5y + 2 \\ x + y < 0 \end{cases}$$

Use the graphing tool to graph the system.

Click to enlarge



To graph $x \geq 5y + 2$

$x - 2 = 5y$

$5y = x - 2$

$y = \frac{1}{2}x - \frac{2}{5}$ ← cannot be graphed on MathXL.

Use techniques from class today:

$5y = x - 2$

choose x so that $x - 2$ is a multiple of 5.

$x = 7$ $\quad 5y = 7 - 2$

$5y = 5$

$y = 1$ $\quad (7, 1)$ plot this, then use slope $\frac{1}{2}$.

Test $(0,0)$

$0 \geq 5(0) + 2$

$0 \geq 2$

false. $(0,0)$ is not a solution. Shade down

$x + y < 0$

Graph $y = -x$ with dotted line.

$(0,0)$ is on this line. Choose a different test point.

$(1,0)$ is not on the line. Test $(1,0)$

$1 + 0 < 0$

$1 < 0$ false. $(1,0)$ is not a solution. shade down.

Shade overlapping shaded areas.